Applied Data Science M.Sc.: Outline for the Pre-Course Mathematics

October 16 - 20, 2023, 9:00-12:30, Bernhard Schmitzer, Uni Göttingen

There will be 2x 90min sessions each morning with 30mins break (=10 sessions), and a problem sheet for Monday to Thursday afternoon. We will not focus on proofs and derivations but on good intuition and practical working knowledge. Tentative list of topics:

- analysis and calculus (2 sessions)
  - limits
  - 1d: derivative, integral, numerical methods
  - higher dimensions: partial derivatives, gradients, Jacobian (use in chain rule and back propagation), volume integral
  - Taylor expansion
  - simple ODEs (exponential growth, harmonic oscillator, flow in vector field), numerical methods, intuition for SDEs
- linear algebra (3 sessions)
  - matrix-vector multiplication, interpretation
  - linear maps, simple examples: rotations, reflections, projections
  - eigenvalues and -vectors, diagonalization
  - matrix inverses
  - hyperplanes, matrix rank, linear systems
  - Cauchy Schwarz inequality
- probabilities and statistics (3 sessions)
  - 1d: discrete distributions (coin toss, binomial,...) , normal distribution, general continuous distributions
  - cumulative distribution functions
  - mean/expectation, standard deviation
  - empirical distributions, estimators for mean and standard deviation
  - conditional probability, Bayes' rule, prior and posterior
  - higher dimensions: covariance, correlation
  - hypothesis testing
  - change of variables for probability distributions (in 1d and higher dimensions)
  - Jensen's inequality
- optimization (2 sessions)
  - motivation / examples (model fitting, MLE, MAP, machine learning, engineering, logistics, ...)
  - simple optimality conditions
  - Lagrange multipliers
  - convexity
  - gradient descent
  - Newton method
  - challenges