Variational Analysis of Empirical Risk Minimization

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This talk presents a variational framework for the asymptotic analysis of empirical risk minimization in general settings. In its most general form the framework concerns a two-stage estimation procedure. In the first stage of the procedure, the trajectory of an observed (but unknown) dynamical system is fit by a trajectory of a known reference dynamical system that minimizes average per-state loss. In the second stage of the procedure, a parameter estimate is obtained from the initial state of the optimal reference trajectory. It turns out that the empirical risk of the best fit trajectory converges almost surely to a constant that can be expressed in variational form as the minimal expected loss over dynamically invariant couplings (joinings) of the observed and reference systems. Moreover, the family of joinings minimizing the expected loss fully characterizes the asymptotic behavior of the estimated parameters. I will illustrate the breadth of the variational framework through applications to the well-studied problems of maximum likelihood estimation and non-linear regression, and will illustrate the breadth of the framework through the analysis of system identification from quantized trajectories subject to noise, a problem in which the models themselves exhibit dynamical behavior across time.