Tax Competition and Equalization: The Impact of Voluntary Cooperation on the Efficiency Goal

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Abstract

Literature has long learned about the welfare improving effect of equalization in tax competition environments. By setting incentives to local authorities, public spending becomes efficient in spite of relying on a mobile resource as the tax base. This paper proves that an equalization system, which is not decided by an autonomous central planner, but in a common decision process of lower-level governments, is not able to internalize the spillovers of tax competition totally. As bargaining requires a certain share of improvements for the regions, a needed compensation within the system results in inefficiencies in the private and the public sector. Additional instruments have to be provided to enhance efficiency. Thereby the constitutional framework to cooperation on equalization can either promote efficiency or equity, a result which is contrary to the literature emphasizing that equalization can imply both.

Keywords: tax competition, fiscal equalization, Nash bargaining, cooperation

JEL-Classification: H10, H71, H77

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1 Introduction

This paper contributes to the discussion on fiscal federalism by analyzing the impact of institutional arrangements on the performance of the public sector. For a long time public finance has set a focus on the efficiency consequences of competition between subordinate jurisdictions in federations. The competition for mobile resources and the accompanying inefficiencies in public spending have been subject to an outpouring of literature.[1] But the welfare loss can be reduced by institutional arrangements. Over the past years research has identified transfer systems that can enhance welfare in competitive environments (Johnson, 1988, Wildasin, 1989, and DePater and Myers, 1994).[2] Whether they are actually effective in reducing the welfare loss depends on the objectives of the implementing government. The traditional approaches assume, that a social planner organizes the public sector to the best of his ability. Equalization systems are then able to internalize the spillovers arising from the competition for mobile resources (Bucovetsky and Smart, 2006, Köthenbürger, 2002). However, welfare maximizing governments are clearly a strong assumption. Büttner et al. (2006) argue that authorities often tend to maximize revenue rather than the social welfare. Consequently equalization sets too strong incentives to raise taxes and an inefficiently high level of public spending is provided. This paper concentrates on the political process determining transfer systems. As regional authorities often gain influence on central policy, equalization is decided in a common decision process of regional authorities rather than by an autonomous central planner.

Transfer systems play an important role in the fiscal relations of many federations. Especially on the local level transfers secure an equal level of public spending among different regions and a minimum supply of public services. The great importance of grants for the local public sector motivates that

[1] See Wilson (1999) and Wilson and Wildasin (2002) for an overview.

[2] This result is confirmed by various empirical studies, e.g., Dahlby and Warren (2003), Büttner (2006), and Karkalakos and Kotsogiannis (2007).

regional authorities aim to influence the shape of the equalization systems to their advantage. Political economy provides different approaches to explain the impact of lower-level governments on central policy decisions concerning transfer systems (e.g., Homburg, 1994, Milligan and Smart, 2005). Either subordinate governments are able to influence upper-level policy in representative chambers or local governments might gain influence more indirectly by lobbyism. Following this local players clearly gain some influence on the central decision concerning transfer systems. Besides, equalization is not only a part of central policy decisions, but it also takes place more indirectly in local cooperative arrangements. Since contributions to local cooperation, like Special Purpose Districts, counties and other forms of local arrangements, are regularly dependent on the local tax base, they mitigate the tax competition like equalization systems do. These jurisdictions are built up for a common public good supply. While a transfer system pays grants in terms of tax revenue to the regions, the allocation of public spending in a local cooperative arrangement is a transfer in terms of public goods. The incentives arising from both institutional arrangements are very similar.

The local impact on centrally organized transfer systems as well as equalization in local cooperative arrangements raises the question whether the local objectives diverge from the central perspective. The decision on equalization is no longer made by one instance, but resembles more a process of bargaining between the authorities of the affiliated jurisdictions. Bargaining as a decision process influence to implemented policy. Therefrom it does seem necessary to rethink the idea of welfare improving equalization systems. Given the large number of functional jurisdictions in many federations the impact of local players on tax competition seems interesting and widely unknown.

This paper provides a bargaining model, in which two regions negotiate on an equalization system. It clarifies not only the priorities local authorities have, when they influence the decision of an upper-level government on fiscal equalization. At the same time it explains the allocation of public spending and the local tax rate decisions in local cooperative arrangements as they are a form of equalization. Even though the instruments provided to cooperation are identical to those of a social planner, efficiency can be neither achieved in the private nor in the public sector. The bargaining process acts like an additional objective to the optimization problem, so that a constitutional framework has to provide more instruments to promote efficiency.

Bargaining in the public sector has been subject to only a few contributions to literature yet. Persson and Tabellini (1996) examine two regions bargaining over a local insurance system against income shocks. They emphasize that regions with better risks will prevent full insurance since this would be redistributing. Kessler et al. (2007) and Hickey (2007) concentrate on cooperation for the sake of internalizing spillovers. However, all these approaches assume the contribution to cooperation to be independent from the local tax rate decision. Looking especially on local cooperative arrangements, there is probably more to the story as contributions often depend on the local tax base. The interaction of tax competition and cooperation is subject to my paper.

The paper is organized as follows. In the next section the basic model is introduced. Then, in section 3, it is extended by an equalization system and its optimal design chosen by a central planner. Section 4 compares this choice with an equalization system negotiated in a common decision process of the regions. The last section concludes the paper.

2 The Model

A federal economy consists of two regions $i, i = 1, 2$. In each region i a representative firm is located. With a mobile factor capital k_i and an immobile factor land, it produces using a linear-homogeneous technology f. Land is equal in both regions, so that the technology can be reduced to a quadratic production function $f(k_i)$. In a competitive market the profit of each firm is zero. The price for capital equals the net rate of return r plus the tax rate τ_i on it. The firms employ capital, so that $f'(k_i) = r + \tau_i$ is satisfied. The capital demand of the regions is expressed by

$$\phi_i(r + \tau_i) = k_i.$$

Since $f''' = 0$ is assumed, $\phi_i'(\tau_i + r)$ is equal in both regions and therefore ϕ'. Furthermore, from the profit maximization condition, it follows $\phi' = \frac{1}{f''} < 0$.

Regions are identical except of their capital endowment s_i. Region 1 provides $s_1 = \frac{S}{2} + \frac{\sigma}{2}$ to the market, $\sigma > 0$, where S is the sum of savings in the federation. At the same time $s_2 = \frac{S}{2} - \frac{\sigma}{2}$ is assumed for region 2, so that the difference of capital endowments between region 1 and 2 is σ. σ is assumed to be small, so that the model deals only with a small asymmetry. The capital stock in the federation is inelastic, so that $S = k_1 + k_2$ holds. Implicit differentiation of this conditions yields

$$\frac{\partial r}{\partial \tau_i} = -\frac{\phi'}{2 \cdot \phi'} = -\frac{1}{2}$$

(proof 1). In both regions a representative resident offers land and capital to the market. Land is paid by a rent π_i, where it is the residual of firms' income and production costs with $\pi_i = f(k_i) - f'(k_i)k_i$. Residents consume a private good c_i and a public good g_i, so that their utility u_i equals

$$u_i = u(c_i, g_i) = c_i + b(g_i) \tag{1}$$

with $b'(g_i) > 0$ and $b''(g_i) < 0$. Since only capital employing firms are taxed, private consumption is a sum of the residual π_i and the interest income rs_i. Hence it is expressed by $c_i = f(k_i) - f'(k_i)k_i + rk_i$.

The public good is offered by a benevolent government, which maximizes residents' utility. To finance public spending a source-based tax τ_i is levied on the regions' employed capital. The budget of the local government must be balanced. It is stated by

$$\tau_i k_i = g_i. \tag{2}$$

In a decentralized economy local governments behave non-cooperatively and compete for capital using the source-based tax as a strategic instrument. Political authorities' optimal choice of the tax rate maximizes residents' utility u_i subject to the local budget:

$$\max_{\tau_i} \quad u_i \quad s.t. \quad \tau_i k_i = g_i. \tag{3}$$

The optimum is described by the first order condition

$$b'(g_i) = \frac{k_i + \frac{1}{2}(s_i - k_i)}{k_i + \frac{1}{2}\tau_i \phi'} \tag{4}$$

(proof 2). While the terms in the numerator constitute the marginal effect of the tax decision on private consumption, the terms in the denominator represent the change of tax revenue. If the capital endowment of the regions is equal, $k_i = s_i$ holds and $b'(g_i) > 1$ is proved. Underprovision of public spending arises in both regions and an inefficient mix of private and public goods is consumed. When local governments increase their tax rates, they generate a capital outflow. Ignoring the positive fiscal externality to the other region, both local governments choose an inefficiently low level of taxation.

If $\sigma > 0$, so that region 1 has a higher capital endowment than region 2 ($s_1 > s_2$), not only fiscal, but pecuniary externalities must be taken into account. As non-price takers on the capital market, local governments influence the net rate of return by their tax policy. A capital exporting region prefers a higher interest rate r than a capital importing region, because the interest income affects private consumption. Since $\frac{\partial r}{\partial \tau_i} < 0$, tax rates influence the net rate of return negatively and a capital exporting region tends to set a lower tax rate in the Nash equilibrium of tax competition. While the pecuniary externality resolves the problem of underprovision at least to some extent in region 2, it is aggregated to the fiscal externality in region 1. As the model deals only with small asymmetries $\bar{\tau}_1 < \bar{\tau}_2$ can be proved for the equilibrium of tax competition (proof 8, result II). It is assumed to be stable. Due to a lower tax rate more capital is employed in region 1 than in region 2 ($\bar{k_1} > \bar{k_2}$). One can expect the local governments to be on the left side of the Laffer curve, so that $\bar{g_1} < \bar{g_2}$ holds. In the Nash equilibrium residents in region 1 have a higher utility than in region 2 ($\bar{u_1} > \bar{u_2}$) (proof 8, result I).

3 Equalization and Tax Competition

Even though fiscal transfer systems aim to equalize public spending in federations, they have also an impact on the local tax decision. Since capital outflow is compensated by higher grants, local governments tend ignore fiscal externalities caused by the tax competition and raise their tax rates. Independent from their original motive cooperative arrangements induce a comparable equalization. Public spending is regularly financed by a contribution on the tax base, so that the impact on the local tax decisions is comparable to a fiscal capacity grant. A small tax base causes a low contribution to the cooperative arrangement, while a region with high fiscal capacity contributes more. Hence the contribution counteracts the tax decision and sets an incentive to raise taxes. While the central planner decides autonomously on a set of contribution rates and transfers $(\vartheta_i, y_i), i = 1, 2$, equalization within cooperative arrangements requires a joint decision of the local governments on these instruments.

Even though different forms of equalization can be observed in federations all over the word, the systems have a basic structure in common. To finance a lump sum transfer y_i, a marginal contribution rate ϑ_i on the local tax base k_i is determined. Therefore the local budget is given by

$$(\tau_i - \vartheta_i)k_i + y_i = g_i. \tag{5}$$

Local governments choose the tax rate in order to maximize residents' utility u_i given in equation (1) so as to balance their budget. The first order condition describes the local tax rate choice in the optimum:

$$b'(g_i) = \frac{k_i + \frac{1}{2}(s_i - k_i)}{k_i + \frac{1}{2}(\tau_i - \vartheta_i)\phi'} \tag{6}$$

(proof 2). Since ϑ_i enters the denominator, the costs for public spending in terms of outflowing capital are reduced. With a raising contribution rate the importance of the fiscal externality for the tax rate choice decreases and the local government raises its tax rate.

Independent from the optimal choice in different institutional settings, equalization gives the opportunity to reallocate welfare. Since tax revenue can be shifted from one region to the other by a lump sum transfer, local budgets are not relevant, but an overall budget of the public sector (proof 3). Therefore both, the central planner as well as the cooperative arrangement, have to balance the budget given by

$$\tau_1 k_1 + \tau_2 k_2 - g_1 - g_2 = 0. \tag{7}$$

All Pareto efficient allocations generated by the equalization system are located on the Pareto frontier P. It is defined by the maximal utility u_2 subject to the budget (7) and a constant level of the utility u_1. The optimization problem yields all allocations attainable by equalization. The choice of instruments, which implements these allocations, can be identified in a second step by the equations (5) and (6). Thus the maximization problem is inverted by choosing first the optimal allocations and then identifying the instruments required to implement this allocations. It is stated by

$$\max_{\tau_i, g_i} \quad u_2 \quad s.t. \quad \tau_1 k_1 + \tau_2 k_2 - g_1 - g_2 = 0 \quad and \quad u_1 - \hat{u_1} = 0 \tag{8}$$

with $\hat{u_1}$ for a given level of utility u_1. If μ_P is the Lagrange operator of the second side order, the slope of the Pareto frontier P is expressed by

$$\frac{du_2^P}{d\hat{u}_1} = -\mu_P \tag{9}$$

(proof 4). The first order conditions also reveal $\mu_P = b'(g_2)/b'(g_1)$. The slope of the Pareto frontier is determined by the marginal utilitiy ratio of public spending in the regions. Besides, total differentiation of all first order conditions of (8) yields

$$\frac{d\mu}{d\hat{u}_1} > 0$$

by Cramers' rule if $1/b'(g_1) + 1/b'(g_2) < 4$ is satisfied (proof 4). Thus the Pareto frontier is strictly concave if the absolute values of the marginal utilities are sufficiently large. Since the analysis concentrates on settings with small asymmetries, one can expect the level of regions' public spending in the Nash bargaining solution to be similar and nearby efficient. Therefore this restriction is not relevant for the Nash bargaining and strict concavity of P can be assumed.

As a benchmark case the allocation chosen by a central government is analyzed. It maximizes the social welfare $w = u_1 + u_2$ by deciding the on local tax rates and the allocation of public spending $(\tau_i, g_i), i = 1, 2$, while the public budget (7) is balanced:

$$\max_{\tau_i, g_i} \quad u_1 + u_2 \quad s.t. \quad \tau_1 k_1 + \tau_2 k_2 - g_1 - g_2 = 0. \tag{10}$$

The optimum is described by the first order conditions $b'(g_1) = b'(g_2)$ and

$$b'(g_i) = \frac{k_i}{k_i + \frac{1}{2}(\tau_i - \tau_j)\phi'} \tag{11}$$

for $i \neq j$ (proof 5). This can only be true if $b'(g_i) = 1, i = 1, 2$, is the case. Hence, it proves $\tau_1 = \tau_2 = \tau^*$. A central government equalizes public spending and the tax rates on an efficient level. Thus capital is equally employed among the regions ($k_1 = k_2 = k^*$). The marginal utilities of public spending and of private consumption are equal within and among the regions. The maximal production output is realized and $\dfrac{\partial u_i}{\partial c_i} = b'(g_i) = 1, i = 1, 2,$ proves first best efficiency to be realized. This result is independent from the capital endowments of the regions.

Concerning the unlike signs of the pecuniary externalities, the incentives set by the central planner are different for both regions. Choosing $\vartheta_i = \tau_i$, the fiscal externalities are reduced to zero. In the case of a capital importing region this contribution rate is too high, because the pecuniary externality counteracts the effect of the fiscal one. At the same time region 1 needs a stronger incentive to raise its tax rate up to an efficient level. To define the instruments chosen by the central planner, consider that the marginal utilities in the equations (6) and (11) are identical and both equal one. From there

$$\frac{1}{2}(s_i - k_i^*) = \frac{1}{2}(\tau_i^* - \vartheta_i^*)\phi' \tag{12}$$

must hold. Equal capital employment among the regions implies that $\frac{1}{2}(s_i - k_i)$ is positive in region 1 but negative in region 2. Hence $(\tau_i - \vartheta_i)$ must be negative for 1 but positive for 2. Thus $\tau_1^* < \vartheta_1^*$ and $\tau_2^* > \vartheta_2^*$ must be satisfied. A lower level of tax rates leads to a higher net rate of return, so that the capital exporter is less willing to increase taxes. An equalization system therefore needs to set stronger incentives to capital exporters than to importers.[3] The incentive is even that strong, that region 1 contributes more to the equalization system than its original tax revenue. It is only able to do this, because it receives a lump sum transfer. The transfers are defined by equation (5). Since g_i and $\tau_i k_i$ are equal for both regions and $\vartheta_1^* > \vartheta_2^*$ holds, $y_1^* > y_2^*$ is proved.

Proposition 1 *The central planner chooses to equalize public spending ($b'(g_i) = 1, i = 1, 2$) as well as the tax rates ($\tau_1^* = \tau_2^*$). Since the capital exporter tends*

[3] If $\sigma = 0$, the result from Bucovetsky and Smart (2006) holds: A full equalizing system internalizes the fiscal externality ($\vartheta_i = \tau_i$). Since no pecuniary externality arises in this situation, first best efficiency is realized.

to set lower tax rates to ensure a higher interest income for its resident, the incentive for region 1 needs to be stronger, so that $\vartheta_1^* > \tau_1^* = \tau_2^* > \vartheta_2^*$ holds.

The central planners' choice of equalization implements $W(u_1^*, u_2^*) = W^*$. The level of social welfare in W^* is the benchmark for the result of the bargaining process between the regions.

4 Bargaining Process

Cooperation differs from other forms of governments at least in two aspects. First of all, it is normally built up voluntarily, so that each affiliated jurisdiction needs to achieve improvement by the arrangement. Even though in cooperative arrangements a common decision process takes place, all partners involved have to accept the decision autonomously. Secondly, one can generally expect the number of players involved in cooperation to be small. Literature argues that larger unions tend to have a greater mismatch of preferences, so that costs of agreements raise with the number of cooperating parties (Alesina et al., 2005). In contrast to a decision process with a large number of participants, a small number enables each government to ensure a certain impact on the policy outcome. Both features of cooperative arrangements imply a decision process, which is more a bargaining between affiliated jurisdictions rather than social welfare maximization within a region. This must be considered in the analysis.

The bargaining process is modeled as a Nash bargaining game. The regions negotiate on the allocation N defined by certain amounts of public spending and corresponding tax rates $(g_i^N, \tau_i^N), i = 1, 2$. Again, the instruments $(\vartheta_i^N, y_i^N), i = 1, 2$, which are required to reinforce the allocation $N(g_i^N, \tau_i^N)$, are identified in a second step, so that the optimization problem is inverted. The cooperative arrangement maximizes the Nash product subject to the overall budget of the public sector given in equation (7). Therefore the optimization problem is stated as follows

$$\max_{\tau_i, g_i} \ n \quad s.t. \quad \tau_1 k_1 + \tau_2 k_2 - g_1 - g_2 = 0 \tag{13}$$

with $n = (u_1 - \bar{u_1})(u_2 - \bar{u_2})$. $\bar{u_i}$ is regions' utility in the disagreement outcome. It is defined as the equilibrium of tax competition without equalization, so that $\bar{u_1} > \bar{u_2}$ holds.

The optimal choice of (13) is described by first order condition

$$\frac{u_1^N - \bar{u_1}}{u_2^N - \bar{u_2}} = \frac{b'(g_1)}{b'(g_2)} := \alpha \tag{14}$$

(proof 6). The ratio of marginal utility of public spending equals the ratio of the absolute improvement the regions achieve from cooperation.

Furthermore, differentiation of (13) yields

$$b'(g_1) = \frac{k_1 + \frac{1}{2}(1-\alpha)(s_1 - k_1)}{k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi'} \tag{15}$$

for the marginal utility of public spending in region 1 and

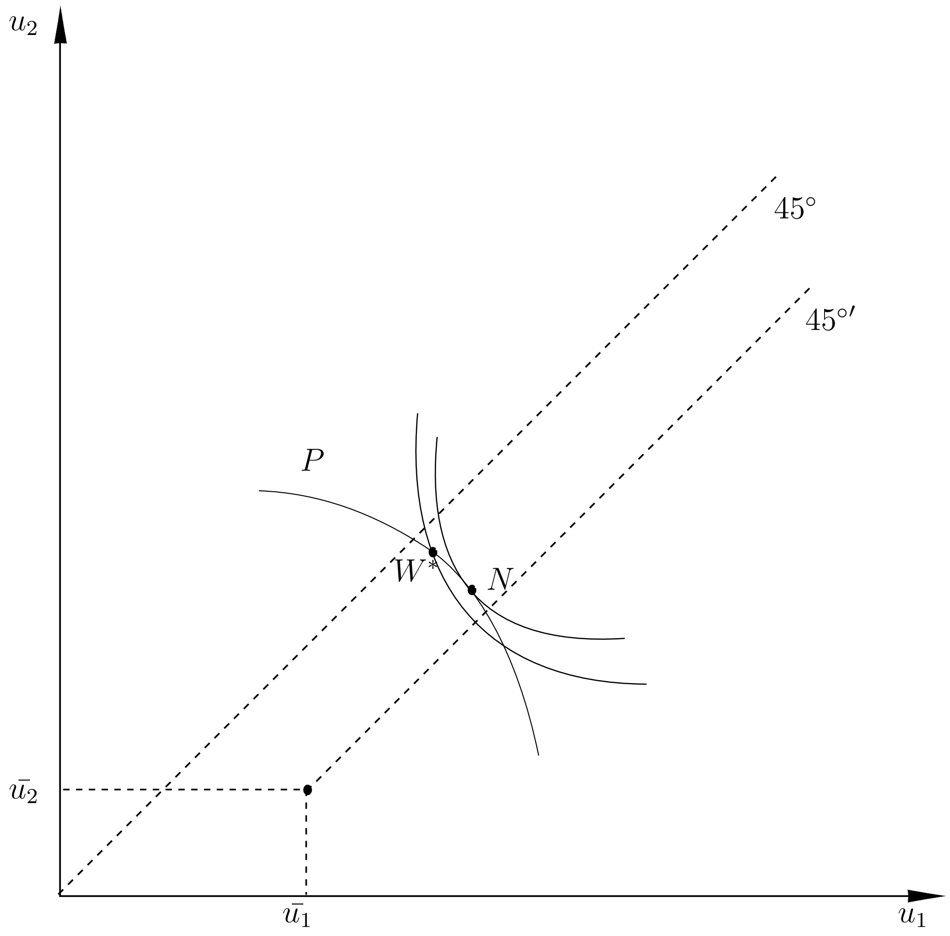
$$b'(g_2) = \frac{k_2 + \frac{1}{2}(\frac{1-\alpha}{\alpha})(s_1 - k_1)}{k_2 - \frac{1}{2}(\tau_1 - \tau_2)\phi'} \tag{16}$$

for the marginal utility of public spending in region 2 (proof 6).

In the Nash bargaining solution N the slope of the Nash product n must be equal to the slope of the Pareto frontier, so that $\frac{du_2^P}{d\hat{u_1}}\big|_N = \frac{du_2^n}{du_1}\big|_N$ holds. N is the targeting point of both curves. Equation (9) implies $\frac{du_2^P}{d\hat{u_1}} = -b'(g_2)/b'(g_1)$ for the slope of the Pareto frontier. By the implicit-function theorem one can show that $\frac{du_2^n}{du_1} = -(u_2 - \bar{u_2})/(u_1 - \bar{u_1})$ holds for the slope of the Nash product.

The central planner chooses W^*, so that efficiency is enhanced. Since $b'(g_1^*) =$

Figure 1: Central Planner vs. Cooperation



$b'(g_2^*) = 1$, the slope of the Pareto frontier in W^* is $-\mu_P = -1$. The slope of the Nash product $\frac{du_2^n}{du_1}$ in the allocation $n(u_1^*, u_2^*) = W^*$ determines the Nash bargaining solution. If $\frac{du_2^n}{du_1}\big|_{W^*} = -1$, the regions agree on the allocation the central planner chooses. For $\frac{du_2^n}{du_1}\big|_{W^*} > -1$ ($\frac{du_2^n}{du_1}\big|_{W^*} < -1$), N differs from the first best solution since the targeting point of the Nash product and the Pareto frontier is on the left (right) side of W^*, so that the Nash product n subtends the Pareto frontier in W^*. One can show, that in W^* the slope of the Nash product is

$$\left.\frac{du_2^n}{du_1}\right|_{W^*} = -\frac{u_2^* - \bar{u_2}}{u_1^* - \bar{u_1}} < -1$$

(proof 8, result III). Hence, from the strict convexity of n (proof 7) it follows that N is an allocation on the right side of W^*. Because of the strict concavity of P and $-\mu = -b'(g_2)/b'(g_1)$, all allocations of the Pareto frontier on the right sight of W^* imply $g_1 > g_2$. At the same time the first order condition (14) of the Nash product comprise $g_1 > g_2$ for all allocations above 45°' line, while below this line the ratio is inverted. Therefrom the Nash bargaining solution N is an allocation on the right sight of W^*, but above the 45°' line. Otherwise the allocation of public services implied by the Pareto frontier and the Nash bargaining solution can not be true at the same time.

Thus $\alpha < 1$ is proved. Public spending is not equalized and smaller in region 2 ($g_1^N > g_2^N$). At the same time the absolute improvement $u_i^N - \bar{u_i}$ is larger for region 2 than for region 1.

Proposition 2 *In a cooperation public spending is not equalized among the regions and $g_1^N > g_2^N$ holds. At the same time region 2 gains more absolute improvement than region 1, so that $(u_1 - \bar{u_1}) < (u_2 - \bar{u_2})$.*

For $\alpha < 1$ the first order conditions of the Nash product determine the tax rate ratio to be $\tau_1^N < \tau_2^N$ (proof 9). Since a higher tax rate results in less capital employment, $k_1^N > k_2^N$ holds.

Proposition 3 *In the Nash bargaining solution $N(u_1^N, u_2^N)$ $\tau_1^N < \tau_2^N$ holds. The private sector is therefore inefficient, since reallocation of capital will cause a higher production output.*

The implementation of the Nash bargaining solution requires a certain choice of instruments $(\vartheta_i^N, y_i^N), i = 1, 2$. However, without further information on the size of the difference in the capital employment, these instruments cannot be compared with the one of the central planner.

The reason for arising inefficiencies in the bargaining solution can be seen in the twofold objective of the bargaining process: On the one hand the equalization system aims to enhance efficiency. On the other hand the bargaining process itself requires a specific share of improvements by the regions. The bargaining solution is in need of acceptance of both local governments. Thinking of the allocation W^* first best efficiency can be achieved by equalization. But since the rich region must be compensated for lower interest income, the equalization system is used to reallocate tax revenue from region 2 to 1. The net transfers provokes a reaction on the capital market, so that $\tau_1^N < \tau_2^N$ holds in N. Therefore region 1 does not only provide a higher level of public spending, the firms in this regions also employ more capital. But still, the region with a better disagreement outcome position is not able to derive the same gain from cooperation.

The model in this paper restricts compensation to transfers between the public sectors. Thus the inefficiency is caused by the lack of instruments. If a transfer between the private sectors would be allowed, this problem could be healed by compensating region 1' decreasing interest income caused by higher tax rates. As compensation is ruled in the private sector, the regions will agree on an efficient allocation of public spending and overall efficiency is achieved. But since the private transfer would reallocate income from the region with less capital endowment to the one with a higher stock of savings, it is hard to think of political acceptance for an instrument like this and the restriction to compensation within the public sector seems to be plausible.

5 Conclusion

The impact of local players on equalization confines its ability to improve social welfare. In a locally decided transfer system public spending is not equalized within the regions and the capital employment is divergent. While in the uncoordinated equilibrium of tax competition the supply of public services is higher in the region with low capital endowment, the equalization system ensures a higher level of public spending in the region with high capital endowment.

Since compensation is restricted to a transfer in terms of public goods, it always entails inefficiency. If a transfer between the private sectors would be allowed, this problem could be healed by compensating region 1' decreasing interest income caused by higher tax rates. Efficiency could be achieved. But since this transfer would reallocate private income from the region with less capital endowment to the one with a higher stock of savings, it is hard to think of political acceptance for an instrument like this. Hence region 1 is compensated by public spending. However, the social welfare is improved by the coordinative character of the jointly implemented equalization system.

The bargaining process acts like an additional objective to the optimization problem. This is the reason why the cooperative arrangement is not able to generate the same level of social welfare like a central planner does, even though the same instruments are available. Therefore a constitutional framework requires a rich set of instruments to promote efficiency. The results of the analysis also suggest that in cooperative arrangements equity and efficiency can not be reached at the same time. While a restriction of fiscal instruments improves the regions' equity to some extent, because the region with a lower utility in the disagreement outcome is able to generate a higher absolute improvement, the efficiency deceases. In contrast, a rich set of instruments allows enhancing efficiency, but, at the same time, the absolute improvement of both regions is equal and equity is not promoted at all. The choice of the constitutional framework for cooperation, made by an upper-level government, has to weight these two contracting objectives. Equity can be realized at the expense of efficiency in the public sector. This is quite a surprising result because literature emphasizes that equalization systems can implement both at the same time: equity and efficiency.

Appendix

Proof 1: Net rate of Return. Note that the net rate of return depends on the vector of local tax rates, so that $r = r(\tau_1, \tau_2)$. Furthermore, consider that the capital market is closed:

$$\phi_1 + \phi_2 = s_1 + s_2.$$

Implicit differentiation of this condition with respect to the tax rate τ_i yields

$$\phi_i'(\tau_i + r)\left(1 + \frac{\partial r}{\partial \tau_i}\right) + \phi_j'(\tau_j + r)\frac{\partial r}{\partial \tau_i} = 0.$$

With $\phi_i'(\tau_i + r) = \phi_j'(\tau_j + r) = \phi'$ it follows

$$\begin{aligned} &\phi' + \phi'\frac{\partial r}{\partial \tau_i} + \phi'\frac{\partial r}{\partial \tau_i} = 0 \\ \Leftrightarrow &2 \cdot \frac{\partial r}{\partial \tau_i}\phi' = -\phi \\ \Leftrightarrow &\frac{\partial r}{\partial \tau_i} = -\frac{1}{2}. \end{aligned}$$

Proof 2: Marginal Utility of the Residents. The derivative of residents' utility depends on the perspective of the treated government. Local authorities take equalization as given and maximize the local utility by choosing an optimal local tax rate. Given the instruments of equalization the tax rate choice determines the level of public spending. From a central perspective the tax rates as well as the level of public spending can be stated autonomously. Reallocation allows to ignore the restriction of local budgets (see proof 3). Thus a tax rate choice does not imply a certain level of public spending.

Local perspective: In a federation without any equalization the utility is given by

$$\begin{aligned}u_i = f(\phi(r(\tau_i,\tau_j)+\tau_i)) - f'\phi(r(\tau_i,\tau_j)+\tau_i) + s_i r(\tau_i,\tau_j) \\ +b(\tau_i\phi(r(\tau_i,\tau_j))) = u_i(\tau_i,\tau_j).\end{aligned}$$

Thus the utility of the resident in region i in an autonomous setting depends on the vector of tax rates of the federation. A local government maximizes residents' utility by choosing the optimal tax rate. The tax revenue then defines local public spending. The local choice is described by

$$\begin{aligned}\frac{\partial u_i}{\partial \tau_i} = f'(k_i)\phi'\left(1+\frac{\partial r}{\partial \tau_i}\right) - f''(k_i)\phi'\left(1+\frac{\partial r}{\partial \tau_i}\right)k_i - f'(k_i)\phi'\left(1+\frac{\partial r}{\partial \tau_i}\right) - s_i\frac{\partial r}{\partial \tau_i} \\ +b'(g_i)\left(k_i+\tau_i\phi'\left(1+\frac{\partial r}{\partial \tau_i}\right)\right) =: 0.\end{aligned}$$

Since $1/f''(k_i) = \phi'$ must be true from profit maximization and $\frac{\partial r}{\partial \tau_i} = -\frac{1}{2}$ holds, rearranging brings

$$\begin{aligned}&-k_i - \frac{1}{2}(s_i-k_i) + b'(g_i)\left(k_i+\tau_i\phi'\left(1-\frac{1}{2}\right)\right) = 0 \\ &\Leftrightarrow b'(g_i) = \frac{k_i+\frac{1}{2}(s_i-k_i)}{k_i+\frac{1}{2}\tau_i\phi'}.\end{aligned}$$

In a federation with equalization a local government receives a lump sum transfer y_i and contributions $\vartheta_i k_i$ to the system. Thus residents' utility is given by

$$\begin{aligned}u_i = f(\phi(r(\tau_i+\tau_j),\tau_i)) - f'\phi(r(\tau_i+\tau_j),\tau_i) + s_i r(\tau_i+\tau_j) \\ +b((\tau_i-\vartheta_i)\phi(r(\tau_i+\tau_j),\tau_i)+y_i) = u_i(\tau_i,\tau_j,\vartheta_i,y_i)\end{aligned}$$

with $i \neq j$. Local authorities maximizes residents' utility by choosing their tax rate τ_i subject to their budget given in equation (5). From the local perspective the instruments of the equalization systems are taken as given, so that the first order condition is

$$\begin{aligned}
\frac{\partial u_i}{\partial \tau_i} &= -k_i - \frac{1}{2}(s_i - k_i) + b'(g_i)\left(k_i + (\tau_i - \vartheta_i)\phi'\left(1 - \frac{1}{2}\right)\right) =: 0 \\
\Leftrightarrow b'(g_i) &= \frac{k_i + \frac{1}{2}(s_i - k_i)}{k_i + \frac{1}{2}(\tau_i - \vartheta_i)\phi'}.
\end{aligned}$$

Central perspective: A central government and a cooperative arrangement are able to reallocate tax revenue by equalization. Thus public spending in the regions is independent from the tax rate. The utility of a resident in region i is stated by

$$u_i = f(\phi(r(\tau_i, \tau_j) + \tau_i)) - f'\phi(r(\tau_i, \tau_j) + \tau_i) + s_i r(\tau_i, \tau_j) + b(g_i) = u_i(\tau_i, \tau_j, g_i) \tag{A.1}$$

with $i \neq j$. Differentiation of equation (A.1) subject to τ_i leads to

$$\begin{aligned}
\frac{\partial u_i}{\partial \tau_i} &= -k_i\left(1 + \frac{\partial r}{\partial \tau_i}\right) + s_i \frac{\partial r}{\partial \tau_i} \\
&= -k_i - \frac{1}{2}(s_i - k_i).
\end{aligned}$$

Due to the mobile tax base, the utility in a region i changes by a tax decision of region j:

$$\begin{aligned}
\frac{\partial u_i}{\partial \tau_j} &= f'(k_i)\phi'\left(1 + \frac{\partial r}{\partial \tau_j}\right) - (r + \tau_i)\phi' \frac{\partial r}{\partial \tau_j} - k_i \frac{\partial r}{\partial \tau_j} + s_i \frac{\partial r}{\partial \tau_j} \\
&= -\frac{1}{2}(s_i - k_i).
\end{aligned}$$

Proof 3: Local and Central Budgets. The local budgets under equalization are given by

$$(\tau_1 - \vartheta_1)k_1 + y_1 = g_1 \tag{A.2}$$
$$(\tau_2 - \vartheta_2)k_2 + y_2 = g_2. \tag{A.3}$$

Adding the equations (A.2) and (A.3) yields

$$\tau_1 k_1 + \tau_2 k_2 + y_1 + y_2 - \vartheta_1 k_1 - \vartheta_1 k_1 = g_1 + g_2.$$

With $\vartheta_1 k_1 + \vartheta_1 k_1 = y_1 + y_2$ for the budget of the transfer system, it follows

$$T = \tau_1 k_1 + \tau_2 k_2 = g_1 + g_2.$$

Proof 4: The Pareto Frontier. The maximization problem describing the Pareto frontier is stated by

$$V_2 = u_2 + \lambda_p(\tau_1 k_1 + \tau_2 k_2 - g_1 - g_2) + \mu_P(u_1 - \hat{u_1}). \tag{A.4}$$

The slope of the Pareto frontier P is $\frac{du_2}{d\hat{u_1}}$. By the Envelope theorem differentiation yields

$$\frac{du_2}{d\hat{u_1}} = -\mu_P \tag{A.5}$$

as the slope of the Pareto frontier P. P is strictly concave if $\frac{d\mu}{d\hat{u_1}} > 0$. In a first step the first order conditions of (A.4) are determined. Note, that the marginal utilities $\frac{\partial u_i}{\partial \tau_j}$ under equalization are given in proof 2. The partial derivative of T with respect to a tax rate is

$$\begin{aligned}\frac{\partial T}{\partial \tau_i} &= k_i + \tau_i \phi' \left(1 + \frac{\partial r}{\partial \tau_i}\right) + \tau_j \phi' \frac{\partial r}{\partial \tau_i} \\ &= k_i + \frac{1}{2}(\tau_i - \tau_j)\phi'\end{aligned}$$

for $i \neq j$. From there the first order conditions of (A.4) describing the Pareto frontier P are:

$$\frac{\partial V_2}{\partial \tau_1} = -\frac{1}{2}(s_2 - k_2) + \lambda_P \left(k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi'\right) + \mu_P \left(-k_1 - \frac{1}{2}(s_1 - k_1)\right) =: 0 \tag{A.6}$$

$$\frac{\partial V_2}{\partial \tau_2} = -k_2 - \frac{1}{2}(s_2 - k_2) + \lambda_P \left(k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi'\right) - \frac{1}{2}\mu_P(s_1 - k_1) =: 0 \tag{A.7}$$

$$\frac{\partial V_2}{\partial g_1} = -\lambda_P + \mu_P b'(g_1) =: 0 \tag{A.8}$$

$$\frac{\partial V_2}{\partial g_2} = b'(g_2) - \lambda_P =: 0 \tag{A.9}$$

$$\frac{\partial V_2}{\partial \lambda_P} = \tau_1 k_1 + \tau_2 k_2 - g_1 - g_2 =: 0 \tag{A.10}$$

$$\frac{\partial V_2}{\partial \mu_P} = u_1 - \hat{u_1} =: 0 \tag{A.11}$$

$\frac{\partial V_2}{\partial v}$ is also written as V_v. Rearranging equation (A.8) and (A.9) leads to $\mu_P = b'(g_2)/b'(g_1)$. Total differentiation of the equation (A.6) to (A.11) leads to a linear equation system, which is written in the matrix notation:

$$\begin{bmatrix} V_{\tau_1,\tau_1} & V_{\tau_1,\tau_2} & V_{\tau_1,g_1} & V_{\tau_1,g_2} & V_{\tau_1,\lambda_P} & V_{\tau_1,\mu_P} \\ V_{\tau_2,\tau_1} & V_{\tau_2,\tau_2} & V_{\tau_2,g_1} & V_{\tau_2,g_2} & V_{\tau_2,\lambda_P} & V_{\tau_2,\mu_P} \\ V_{g_1,\tau_1} & V_{g_1,\tau_2} & V_{g_1,g_1} & V_{g_1,g_2} & V_{g_1,\lambda_P} & V_{g_1,\mu_P} \\ V_{g_2,\tau_1} & V_{g_2,\tau_2} & V_{g_2,g_1} & V_{g_2,g_2} & V_{g_2,\lambda_P} & V_{g_2,\mu_P} \\ V_{\lambda_p,\tau_1} & V_{\lambda_P,\tau_2} & V_{\lambda_P,g_1} & V_{\lambda_P,g_2} & V_{\lambda_P,\lambda_P} & V_{\lambda_P,\mu_P} \\ V_{\mu_P,\tau_1} & V_{\mu_P,\tau_2} & V_{\mu_P,g_1} & V_{\mu_P,g_2} & V_{\mu_P,\lambda_P} & V_{\mu_P,\mu_P} \end{bmatrix} \cdot \begin{bmatrix} d\tau_1 \\ d\tau_2 \\ dg_1 \\ dg_2 \\ d\lambda_P \\ d\mu_P \end{bmatrix} = \begin{bmatrix} -V_{\tau_1,\hat{u_1}} \\ -V_{\tau_2,\hat{u_1}} \\ -V_{g_1,\hat{u_1}} \\ -V_{g_2,\hat{u_1}} \\ -V_{\lambda_P,\hat{u_1}} \\ -V_{\mu_P,\hat{u_1}} \end{bmatrix} d\hat{u_1} \tag{A.12}$$

$V_{v,w}$ is obtained by differentiating the derivatives $V_v, v = g_1, ..., \mu_P$ with respect to the variables $w, w = g_1, ..., \mu_P$. The matrix on the left hand side of equation (A.12) is labeled with D. By Cramer's rule, $d\mu = \frac{|D^\mu|}{|D|}$ expresses the solution for marginal change of μ.

$|D|$ is a Bordered Hessian determinant. As equation (A.4) has two side conditions, $|D|$ has two borders ($m = 2$). Furthermore, (A.4) provides the four choice variables g_1, g_2, τ_1 and τ_2, so that its number is $n = 4$. Since the sign of a matrix with $n = m + 2$ is $(-1)^{m+2}$, $|D| > 0$ holds (Chiang, 2005: 362).

$|D^\mu|$ is the determinant of the matrix D^μ, which is obtained by replacing the last column of $|D|$ by the right hand side of (A.12). $|D^\mu|$ is stated by

$$|D^\mu| = \begin{vmatrix} -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & 0 & 0 & k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi' & 0 \\ \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & 0 & 0 & k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi' & 0 \\ 0 & 0 & \mu_P b''(g_1) & 0 & -1 & 0 \\ 0 & 0 & 0 & b''(g_2) & -1 & 0 \\ k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi' & k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi' & -1 & -1 & 0 & 0 \\ -k_1 - \frac{1}{2}(s_1 - k_1) & -\frac{1}{2}(s_1 - k_1) & b'(g_1) & 0 & 0 & d\hat{u}_1 \end{vmatrix}.$$

$|D^\mu|$ is expanded by the last column. Deleting the last row and the last column leads to the minor $|D^{\mu'}|$ of the element $d\hat{u}_1$, so that

$$|D^\mu| = |D^\mu_{6,6}|d\hat{u}_1$$

is true. The minor $|D^\mu_{6,6}|$ is given by

$$|D^\mu_{6,6}| = \begin{vmatrix} -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & 0 & 0 & k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi' \\ \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & 0 & 0 & k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi' \\ 0 & 0 & \mu_P b''(g_1) & 0 & -1 \\ 0 & 0 & 0 & b''(g_2) & -1 \\ k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi' & k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi' & -1 & -1 & 0 \end{vmatrix}.$$

$|D^\mu_{6,6}|$ is expanded by the forth column. Thus it is given by

$$|D^\mu_{6,6}| = b''(g_2)|D^{\mu'}_{4,4}| - (-1)|D^{\mu'}_{5,4}|. \tag{A.13}$$

$|D^{\mu'}_{4,4}|$ is the minor to the element $b''(g_2)$ and obtained by deleting the forth row and the forth column of $|D^{\mu}_{6,6}|$, so that it equates

$$|D^{\mu'}_{4,4}| = \begin{vmatrix} -\phi'(\frac{1}{4}-\mu_P\lambda_P+\frac{1}{4}\mu_P) & \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & 0 & k_1+\frac{1}{2}(\tau_1-\tau_2)\phi' \\ \phi'(\frac{1}{4}-\mu_P\lambda_P+\frac{1}{4}\mu_P) & -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & 0 & k_2+\frac{1}{2}(\tau_2-\tau_1)\phi' \\ 0 & 0 & \mu_P b''(g_1) & -1 \\ k_1+\frac{1}{2}(\tau_1-\tau_2)\phi' & k_2+\frac{1}{2}(\tau_2-\tau_1)\phi' & -1 & 0 \end{vmatrix}.$$

$|D^{\mu'}_{4,4}|$ is expanded by the third column, so that

$$|D^{\mu'}_{4,4}| = \mu_P b''(g_1)|D^{\mu''}_{3,3}| - (-1)|D^{\mu''}_{3,4}|$$

holds. To ascertain $|D^{\mu'}_{4,4}|$, $|D^{\mu''}_{3,3}|$ and $|D^{\mu''}_{3,4}|$ are determined. $|D^{\mu''}_{3,3}|$ is obtained by deleting the third row and the third column of $|D^{\mu'}_{4,4}|$, so that

$$|D^{\mu''}_{3,3}| = \begin{vmatrix} -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & k_1+\frac{1}{2}(\tau_1-\tau_2)\phi' \\ \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & k_2+\frac{1}{2}(\tau_2-\tau_1)\phi' \\ k_1+\frac{1}{2}(\tau_1-\tau_2)\phi' & k_2+\frac{1}{2}(\tau_2-\tau_1)\phi' & 0 \end{vmatrix}.$$

$|D^{\mu''}_{3,3}|$ is expanded by the last column. Therefore it yields

$$\begin{aligned} |D^{\mu''}_{3,3}| &= \left(k_1+\frac{1}{2}(\tau_1-\tau_2)\phi'\right) \begin{vmatrix} \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) \\ k_1+\frac{1}{2}(\tau_1-\tau_2)\phi' & k_2+\frac{1}{2}(\tau_2-\tau_1)\phi' \end{vmatrix} \\ &\quad - \left(k_2+\frac{1}{2}(\tau_2-\tau_1)\phi'\right) \begin{vmatrix} -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) \\ k_1+\frac{1}{2}(\tau_1-\tau_2)\phi' & k_2+\frac{1}{2}(\tau_2-\tau_1)\phi \end{vmatrix} \\ &= \left(k_1+\frac{1}{2}(\tau_1-\tau_2)\phi'\right)\phi'\left(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P\right)(k_1+k_2) \\ &\quad + \left(k_2+\frac{1}{2}(\tau_2-\tau_1)\phi'\right)\phi'\left(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P\right)(k_1+k_2) \\ &= \phi'\left(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P\right)(k_1+k_2)^2. \end{aligned}$$

The determinant $|D^{\mu''}_{3,4}|$ as a subdeterminant of $|D^{\mu'}_{4,4}|$ is obtained by deleting the forth row and the third column of the matrix $D^{\mu'}_{4,4}$. Thus it is stated by

$$|D^{\mu''}_{3,4}| = \begin{vmatrix} -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi' \\ \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi' \\ 0 & 0 & -1 \end{vmatrix}.$$

$|D^{\mu''}_{3,4}|$ is expanded by the last row, so that

$$|D^{\mu''}_{3,4}| = (-1)|D^{\mu'''}_{3,3}|$$

holds. $|D^{\mu'''}_{3,3}|$ is the subdeterminant of $|D^{\mu''}_{3,4}|$ and obtained by deleting the last row and the last column. From there

$$|D^{\mu'''}_{3,4}| = (-1)\begin{vmatrix} -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) \\ \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) \end{vmatrix} = 0$$

holds. Inserting $|D^{\mu''}_{3,3}|$ and $|D^{\mu''}_{3,4}| = 0$ in $|D^{\mu'}_{4,4}|$ yields

$$|D^{\mu'}_{4,4}| = \mu_P b''(g_1)\left(\phi'\left(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P\right)\right)(k_1 + k_2)^2.$$

To ascertain $|D^{\mu}_{6,6}|$, $|D^{\mu'}_{5,4}|$ must be defined. $|D^{\mu'}_{5,4}|$ is the subdeterminant of $|D^{\mu}_{6,6}|$ and it is obtained by deleting the fifth row and the forth column:

$$|D^{\mu'}_{5,4}| = \begin{vmatrix} -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & 0 & k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi' \\ \phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & -\phi'(\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P) & 0 & k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi' \\ 0 & 0 & \mu_P b''(g_1) & -1 \\ 0 & 0 & 0 & -1 \end{vmatrix}.$$

The determinant $|D^{\mu'}_{5,4}|$ is expanded by the last row, so that

$$|D^{\mu'}_{5,4}| = (-1)|D^{\mu''}_{4,4}|$$

holds. The minor $|D^{\mu''}_{4,4}|$ of the element (-1) is obtained by deleting the last row and the last column of $|D^{\mu'}_{5,4}|$. Thus

$$|D^{\mu''}_{4,4}| = \begin{vmatrix} -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & 0 \\ \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & 0 \\ 0 & 0 & \mu_P b''(g_1) \end{vmatrix}$$

is true. $|D^{\mu''}_{4,4}|$ is expanded to the element $\mu_P b''(g_1)$ with the minor $|D^{\mu'''}_{3,3}|$, while

$$\begin{aligned} |D^{\mu'''}_{3,3}| &= \begin{vmatrix} -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) \\ \phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) & -\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P) \end{vmatrix} \\ &= 0 \end{aligned}$$

holds. With $|D^{\mu'''}_{3,3}| = 0$, the determinants $|D^{\mu''}_{4,4}|$ and $|D^{\mu'}_{5,4}|$ are proved to be zero. Inserting $|D^{\mu'}_{4,4}|$ and $|D^{\mu'}_{5,4}| = 0$ in equation (A.13) yields

$$|D^{\mu}_{6,6}| = \mu_P b''(g_2) b''(g_1) \left(\phi'\left(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P\right)\right)(k_1+k_2)^2,$$

so that

$$|D^{\mu}| = \mu_P b''(g_2) b''(g_1) \left(\phi'\left(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P\right)\right)(k_1+k_2)^2 d\hat{u}_1$$

is true. Rearranging yields

$$\frac{d\mu_P}{d\hat{u}_1} = \frac{\mu_P b''(g_2) b''(g_1)(\phi'(\frac{1}{4}-\lambda_P+\frac{1}{4}\mu_P))(k_1+k_2)^2}{|D|}.$$

Since $b''(g_i)$ and ϕ' are negative and $|D|$ is positive, $\frac{d\mu_P}{d\hat{u}_1} > 0$ is proved if, and only if, $\frac{1}{4} - \lambda_P + \frac{1}{4}\mu_P < 0$. As $\lambda_P = b'(g_2)$ and $\mu_P = b'(g_2)/b'(g_1)$ hold, this condition can be written as

$$\begin{aligned} &\Rightarrow \frac{1}{4} - b'(g_2) + \frac{1}{4}\frac{b'(g_2)}{b'(g_1)} < 0 \\ &\Leftrightarrow 1 + \frac{b'(g_2)}{b'(g_1)} < 4b'(g_2) \end{aligned}$$

$$\Leftrightarrow \frac{1}{b'(g_2)} + \frac{1}{b'(g_1)} < 4.$$

For all allocations of public spending, which satisfy $\frac{1}{b'(g_2)} + \frac{1}{b'(g_1)} < 4$, $\frac{d\mu}{d\hat{u}_1}$ is positive.

Proof 5: Optimization of the Central Planner. The optimization problem of the central planner is stated by

$$\mathcal{L}_1 = u_1 + u_2 + \lambda_1(\tau_1 k_1 + \tau_2 k_2 - g_1 - g_2). \tag{A.14}$$

The optimum of (A.14) is described by the first order conditions

$$\frac{\partial \mathcal{L}_1}{\partial \tau_1} = -k_1 - \frac{1}{2}(s_1 - k_1) - \frac{1}{2}(s_2 - k_2) + \lambda_1 \left(k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi' \right) =: 0 \tag{A.15}$$

$$\frac{\partial \mathcal{L}_1}{\partial \tau_2} = -\frac{1}{2}(s_1 - k_1) - k_2 - \frac{1}{2}(s_2 - k_2) + \lambda_1 \left(k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi' \right) =: 0 \tag{A.16}$$

$$\frac{\partial \mathcal{L}_1}{\partial g_1} = b'(g_1) - \lambda_1 =: 0 \tag{A.17}$$

$$\frac{\partial \mathcal{L}_1}{\partial g_2} = b'(g_2) - \lambda_1 =: 0 \tag{A.18}$$

From the equations (A.17) and (A.18) it follows $b'(g_1) = b'(g_2) = \lambda_1$. Using also $(s_1 - k_1) = -(s_2 - k_2)$, rearranges the first order conditions (A.15) and (A.16) to

$$b'(g_1) = \frac{k_1}{k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi'} \tag{A.19}$$

$$b'(g_2) = \frac{k_2}{k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi'}. \tag{A.20}$$

Proof 6: The Bargaining Solution. The bargaining game of the cooperative arrangement is stated by

$$n = (u_1 - \bar{u_1})(u_2 + \bar{u_2}) + \lambda_2(\tau_1 k_1 + \tau_2 k_2 - g_1 - g_2).$$

The optimum of the Nash product is given by the first order conditions

$$\frac{\partial n}{\partial \tau_1} = \left[-k_1 - \frac{1}{2}(s_1 - k_1)\right](u_2 - \bar{u_2}) + (u_1 - \bar{u_1})\left[-\frac{1}{2}(s_2 - k_2)\right] \tag{A.21}$$
$$+ \lambda_2\left(k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi'\right) =: 0$$

$$\frac{\partial n}{\partial \tau_2} = \left[-k_2 - \frac{1}{2}(s_2 - k_2)\right](u_1 - \bar{u_1}) + (u_2 - \bar{u_2})\left[-\frac{1}{2}(s_1 - k_1)\right] \tag{A.22}$$
$$+ \lambda_2\left(k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi'\right) =: 0$$

$$\frac{\partial n}{\partial g_1} = b'(g_1)(u_2 - \bar{u_2}) - \lambda_2 =: 0 \tag{A.23}$$

$$\frac{\partial n}{\partial g_2} = b'(g_2)(u_1 - \bar{u_1}) - \lambda_2 =: 0 \tag{A.24}$$

From the equations (A.23) and (A.24) it follows

$$b'(g_1)(u_2 - \bar{u_2}) = b'(g_2)(u_1 - \bar{u_1})$$
$$\Leftrightarrow \frac{b'(g_1)}{b'(g_2)} = \frac{u_1 - \bar{u_1}}{u_2 - \bar{u_2}} =: \alpha \tag{A.25}$$

Furthermore, with $b'(g_i)(u_j - \bar{u_j}) = \lambda_2$ the first order condition (A.21) is given by

$$\left[-k_1 - \frac{1}{2}(s_1 - k_1)\right](u_2 - \bar{u_2}) + (u_1 - \bar{u_1})\left[-\frac{1}{2}(s_2 - k_2)\right]$$
$$+ b'(g_1)(u_2 - \bar{u_2})\left(k_1 + \frac{1}{2}\tau_1\phi'\right) - b'(g_2)(u_1 - \bar{u_1})\left(-\frac{1}{2}\tau_2\phi'\right) = 0$$

$$\Leftrightarrow \left[-k_1 - \frac{1}{2}(s_1 - k_1) + b'(g_1)\left(k_1 + \frac{1}{2}\tau_1\phi'\right)\right](u_2 - \bar{u_2})$$
$$- (u_1 - \bar{u_1})\left[\frac{1}{2}(s_2 - k_2) + b'(g_2)\left(-\frac{1}{2}\tau_1\phi'\right)\right] = 0 \tag{A.26}$$

By virtue of equation (A.25) $(u_1 - \bar{u_1}) = \alpha(u_2 - \bar{u_2})$ holds, so that equation

(A.26) is rewritten as

$$
\begin{aligned}
&\Rightarrow -k_1 - \frac{1}{2}(s_1 - k_1) + b'(g_1)\left(k_1 + \frac{1}{2}\tau_1\phi'\right) + \alpha\frac{1}{2}(s_1 - k_1) - b'(g_1)\left(-\frac{1}{2}\tau_1\phi'\right) = 0 \\
&\Leftrightarrow -k_1 - \frac{1}{2}(1-\alpha)(s_1 - k_1) + b'(g_1)\left(k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi'\right) = 0 \\
&\Leftrightarrow b'(g_1) = \frac{k_1 + \frac{1}{2}(1-\alpha)(s_1 - k_1)}{k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi'}.
\end{aligned}
$$

The first order condition (A.22) can analogously be rearranged with $(u_2 - \bar{u_2}) = \frac{1}{\alpha}(u_1 - \bar{u_1})$ to the marginal utility of public spending in region 2:

$$
b'(g_2) = \frac{k_2 + \frac{1}{2}(\frac{1-\alpha}{\alpha})(s_1 - k_1)}{k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi'}.
$$

Proof 7: Strict Convexity of the Nash Product. The Nash product is given by $n = (u_1 - \bar{u_1})(u_2 - \bar{u_2})$. Strict convexity of n is given if the second derivative is positive. If $\frac{\partial^2 u_2}{\partial u_1^2} > 0$ holds, n is strictly convex. The implicit-function theorem leads to

$$
\frac{du_2}{du_1} = -\frac{u_2 - \bar{u_2}}{u_1 - \bar{u_1}}
$$

for the first derivative. The second derivative is given by

$$
\frac{\partial^2 u_2^n}{\partial u_1^2} = \frac{u_2 - \bar{u_2}}{u_1 - \bar{u_1}}.
$$

Since cooperation is voluntary, both regions have to have to achieve an improvement by the arrangement. Therefore $u_i - \bar{u_i} > 0, i = 1, 2,$ must be true. Thus $\frac{\partial^2 u_2}{\partial u_1^2} > 0$ is proved to be positive. The Nash product is strictly convex for all relevant settings.

Proof 8: Absolute Improvement of the Regions. To prove $u_2^* - \bar{u_2} >$

$u_1^* - \bar{u}_1$, see that

$$\begin{aligned} &u_2^* - \bar{u}_2 > u_1^* - \bar{u}_1 \\ \Leftrightarrow &\bar{u}_1 - \bar{u}_2 > u_1^* - u_2^*. \end{aligned}$$

Since in W^* the allocation of public spending is given with $g_1 = g_2 = g^*$ and $\tau_1 = \tau_2 = \tau^*$ holds for the tax rates, the utilities u_1^* and u_2^* only differ in the interest income:

$$\begin{aligned} \bar{u}_1 - \bar{u}_2 > &f(k^*) - f'k^* + s_1 r^* + b(g^*) - f(k^*) + f'k^* - s_2 r^* - b'(g^*) \\ \Leftrightarrow &\bar{u}_1 - \bar{u}_2 > \sigma r^*. \end{aligned}$$

Since σ and r^* are both positive, residents' utility in the disagreement outcome is higher in region 1 than in region 2. Thus $\bar{u}_1 - \bar{u}_2 > 0$ holds (**result I**). Furthermore, see that

$$\bar{u}_1 - \bar{u}_2 = \int_o^\sigma \frac{d(\bar{u}_1 - \bar{u}_2)}{d\tilde{\sigma}} d\tilde{\sigma}$$

is true. By the Envelope theorem for Nash equilibria (Caputo, 1996) the derivative to the differences in capital endowment σ is stated by

$$\frac{d(\bar{u}_1 - \bar{u}_2)}{d\tilde{\sigma}} = \bar{r} + \frac{\partial \bar{u}_1}{\partial \bar{\tau}_2}\frac{\partial \bar{\tau}_2}{\partial \tilde{\sigma}} - \frac{\partial \bar{u}_2}{\partial \bar{\tau}_1}\frac{\partial \bar{\tau}_1}{\partial \tilde{\sigma}}$$

Let $\frac{\partial \bar{u}_1}{\partial \bar{\tau}_2}\frac{\partial \bar{\tau}_2}{\partial \tilde{\sigma}} - \frac{\partial \bar{u}_2}{\partial \bar{\tau}_1}\frac{\partial \bar{\tau}_1}{\partial \tilde{\sigma}}$ be X. If $X > 0$ holds for all $\tilde{\sigma}$,

$$\frac{d(\bar{u}_1 - \bar{u}_2)}{d\tilde{\sigma}} > \bar{r}$$

is true. Hence $\bar{u}_1 - \bar{u}_2 > \sigma \bar{r}$ is proved with $X > 0$. If, in addition, $\bar{r} > r^*$ holds,

$$\begin{aligned} &\bar{u}_1 - \bar{u}_2 > \sigma \bar{r} > \sigma r^* \\ \Rightarrow &u_2^* - \bar{u}_2 > u_1^* - \bar{u}_1 \end{aligned} \tag{A.27}$$

is proved.

$X > 0$ holds if

$$\frac{\partial \bar{u}_1}{\partial \bar{\tau}_2}, \frac{\partial \bar{\tau}_2}{\partial \tilde{\sigma}}, \frac{\partial \bar{u}_2}{\partial \bar{\tau}_1} > 0$$
$$\frac{\partial \bar{\tau}_1}{\partial \tilde{\sigma}} < 0$$

can be proved. In a first step $\frac{\partial \bar{u}_i}{\partial \bar{\tau}_j} > 0$ is investigated. In a second step the signs of $\frac{\partial \bar{\tau}_i}{\partial \tilde{\sigma}}, i = 1, 2$ are determined. The utility $\bar{u}_i$ in the disagreement outcome is differentiated with respect to $\tau_j, i \neq j$ so that

$$\frac{\partial u_i}{\partial \tau_j} = -\frac{1}{2}(s_i - k_i) - \frac{1}{2} b'(g_i) \tau_i \phi'.$$

The first term on the right hand side is negative if the region is a capital exporter and it is positive if the region is a capital importer. The second term is positive. Since I concentrate on a setting with small asymmetries and therefore $(s_i - k_i)$ is supposed to be small, one can expect that the effect of the second term dominates the first one in the case of a capital exporter. Otherwise region i would prefer a low tax rate in region j. This does not seem to be plausible for small asymmetries, because it benefits from a higher tax rate level by inflowing capital. It follows $\frac{\partial u_i}{\partial \tau_j} > 0$.

To ascertain $\frac{\partial \bar{\tau}_i}{\partial \tilde{\sigma}}$, see that the equation (4) determines the local tax rate choice. With

$$-k_i - \frac{1}{2}(s_i - k_i) + b'(g_i)\left(k_i + \frac{1}{2}\tau_i \phi'\right) = 0 \tag{A.28}$$

for the first order condition in region i,

$$\frac{\partial^2 u_1}{\partial \tau_1 \partial \sigma} = -\frac{1}{4}\phi'\left(\frac{\partial \tau_1}{\partial \sigma} - \frac{\partial \tau_2}{\partial \sigma}\right) - \frac{1}{4} + b''(g_1)\left(k_1 \frac{\partial \tau_1}{\partial \sigma} + \frac{1}{2}\tau_1 \phi'\left(\frac{\partial \tau_1}{\partial \sigma} - \frac{\partial \tau_2}{\partial \sigma}\right)\right)\left(k_1 + \frac{1}{2}\tau_1 \phi'\right)$$
$$+ b'(g_1)\left(\phi' \frac{\partial \tau_1}{\partial \sigma} - \frac{1}{2}\phi' \frac{\partial \tau_2}{\partial \sigma}\right) =: 0$$

holds for region 1. Rewriting yields

$$\underbrace{\left[-\frac{1}{4}\phi' + b''(g_1)\left(k_1 + \frac{1}{2}\tau_1\phi'\right)^2 + b'(g_1)\phi'\right]}_{a}\frac{\partial\tau_1}{\partial\sigma} - \frac{1}{4} \tag{A.29}$$
$$+\underbrace{\left[\frac{1}{4}\phi' - b''(g_1)\frac{1}{2}\tau_1\phi'\left(k_1 + \frac{1}{2}\tau_1\phi'\right) - \frac{1}{2}b'(g_1)\phi'\right]}_{b}\frac{\partial\tau_2}{\partial\sigma} = 0.$$

The first squared bracket is assumed to be a and the second squared bracket is assumed to be b. Thus we can write equation (A.29) as

$$a\cdot\frac{\partial\tau_1}{\partial\sigma} - \frac{1}{4} + b\cdot\frac{\partial\tau_2}{\partial\sigma} = 0$$
$$\Leftrightarrow\frac{\partial\tau_1}{\partial\sigma} - \frac{1}{4a} + \frac{b}{a}\cdot\frac{\partial\tau_2}{\partial\sigma} = 0. \tag{A.30}$$

Differentiating the first order condition (A.28) with respect to σ yields

$$\frac{\partial^2 u_2}{\partial\tau_2\partial\sigma} = -\frac{1}{4}\phi'\left(\frac{\partial\tau_2}{\partial\sigma} - \frac{\partial\tau_1}{\partial\sigma}\right) + \frac{1}{4} + b''(g_2)\left(k_2\frac{\partial\tau_2}{\partial\sigma} + \frac{1}{2}\tau_2\phi'\left(\frac{\partial\tau_2}{\partial\sigma} - \frac{\partial\tau_1}{\partial\sigma}\right)\right)\left(k_2 + \frac{1}{2}\tau_2\phi'\right)$$
$$+b'(g_2)\left(\phi'\frac{\partial\tau_2}{\partial\sigma} - \frac{1}{2}\phi'\frac{\partial\tau_1}{\partial\sigma}\right) =: 0.$$

Rearranging leads to

$$\underbrace{\left[-\frac{1}{4}\phi' + b''(g_2)\left(k_2 + \frac{1}{2}\tau_2\phi'\right)^2 + b'(g_2)\phi'\right]}_{c}\frac{\partial\tau_2}{\partial\sigma} + \frac{1}{4} \tag{A.31}$$
$$+\underbrace{\left[\frac{1}{4}\phi' - b''(g_2)\frac{1}{2}\tau_2\phi'\left(k_2 + \frac{1}{2}\tau_2\phi'\right) - \frac{1}{2}b'(g_2)\phi'\right]}_{d}\frac{\partial\tau_1}{\partial\sigma} = 0.$$

The first squared bracket is assumed to be c and the second squared bracket is assumed to be d. Thus we can write equation (A.31) as

$$c\cdot\frac{\partial\tau_2}{\partial\sigma} + \frac{1}{4} + d\cdot\frac{\partial\tau_1}{\partial\sigma} = 0$$
$$\Leftrightarrow\frac{c}{d}\cdot\frac{\partial\tau_2}{\partial\sigma} + \frac{1}{4d} + \frac{\partial\tau_1}{\partial\sigma} = 0. \tag{A.32}$$

To ascertain the sign of $\frac{\partial\bar{\tau}_2}{\partial\tilde{\sigma}}$ in the tax competition equilibrium, the equation

(A.32) is subtracted from (A.30), so that

$$
\begin{aligned}
&-\frac{1}{4a}-\frac{1}{4d}+\left[\frac{b}{a}-\frac{c}{d}\right]\frac{\partial\tau_2}{\partial\sigma}=0\\
\Leftrightarrow&\frac{\partial\bar{\tau}_2}{\partial\tilde{\sigma}}=-\frac{a+d}{4(ac-bd)}. \qquad \text{(A.33)}
\end{aligned}
$$

To determine $\frac{\partial\bar{\tau}_1}{\partial\tilde{\sigma}}<0$, rearrange the equations (A.30) and (A.32) to

$$
\frac{a}{b}\frac{\partial\tau_1}{\partial\sigma}-\frac{1}{4b}+\frac{\partial\tau_2}{\partial\sigma}=0 \qquad \text{(A.34)}
$$

$$
\frac{d}{c}\frac{\partial\tau_1}{\partial\sigma}+\frac{1}{4c}+\frac{\partial\tau_2}{\partial\sigma}=0. \qquad \text{(A.35)}
$$

Subtraction of (A.35) from (A.34) yields

$$
\frac{\partial\bar{\tau}_1}{\partial\tilde{\sigma}}=\frac{b+c}{4(ac-bd)}. \qquad \text{(A.36)}
$$

I begin by investigating $a+d$. It is stated as

$$
\begin{aligned}
a+d=&\left[-\frac{1}{4}\phi'+b''(g_1)\left(k_1+\frac{1}{2}\tau_1\phi'\right)^2+b'(g_1)\phi'\right]_a\\
&+\left[\frac{1}{4}\phi'-b''(g_2)\frac{1}{2}\tau_2\phi'\left(k_2+\frac{1}{2}\tau_2\phi'\right)-\frac{1}{2}b'(g_2)\phi'\right]_d\\
=&b''(g_1)\left(k_1+\frac{1}{2}\tau_1\phi'\right)^2+b'(g_1)\phi'-b''(g_2)\frac{1}{2}\tau_2\phi'\left(k_2+\frac{1}{2}\tau_2\phi'\right)-\frac{1}{2}b'(g_2)\phi'.
\end{aligned}
$$

(A.37)

$b''(g_i)$ and ϕ' are negative. As the regions are on the left side of the Laffer curve, $k_i+\frac{1}{2}\tau_i\phi'$ is positive. From there the first three terms of (A.37) are negative. The last term of (A.37) is positive. Since the asymmetry is assumed to be small, the sum of the second and the forth term is negative. Hence $a+d<0$ is proved.

Inserting the squared brackets from the equations (A.29) and (A.31) into

$b + c$ yields

$$b + c = \left[-\frac{1}{4}\phi' + b''(g_2)\left(k_2 + \frac{1}{2}\tau_2\phi'\right)^2 + b'(g_2)\phi' \right]_c$$
$$+ \left[\frac{1}{4}\phi' - b''(g_1)\frac{1}{2}\tau_1\phi'\left(k_1 + \frac{1}{2}\tau_1\phi'\right) - \frac{1}{2}b'(g_1)\phi' \right]_b$$
$$= b''(g_2)\left(k_2 + \frac{1}{2}\tau_2\phi'\right)^2 + b'(g_2)\phi' - b''(g_1)\frac{1}{2}\tau_1\phi'\left(k_1 + \frac{1}{2}\tau_1\phi'\right) - \frac{1}{2}b'(g_1)\phi'. \tag{A.38}$$

From what we have learned already, the first and the third term of (A.38) are negative. Furthermore, the sum of $b'(g_2) - \frac{1}{2}b'(g_1)$ is positive if

$$b'(\bar{g}_1) \leq \frac{1}{2}b'(\bar{g}_2) \tag{A.39}$$

holds in the disagreement outcome. As the analysis deals with small asymmetries in the capital market, the local tax decision does not vary a lot among the regions. From there $b + c < 0$ is true.

Last, the term $ac - bd$ must be examined. For that the assumption of a stable equilibrium in tax competition is used. If $ac - bd$ is the determinant of the Jacobian matrix describing the equilibrium of tax competition, it has to be positive (Chiang, 2005: 627). Furthermore, the trace $trcJ$ has to be negative. The local tax rate choice is given in the first order condition $\frac{\partial u_i}{\partial \tau_i} = 0$. Thus the equilibrium of tax competition is described by the Jacobian matrix $|J|$ to these equations. If

$$|J| = \begin{vmatrix} \frac{\partial^2 u_1}{\partial \tau_1^2} & \frac{\partial^2 u_1}{\partial \tau_1 \partial \tau_2} \\ \frac{\partial^2 u_2}{\partial \tau_1 \partial \tau_2} & \frac{\partial^2 u_2}{\partial \tau_2^2} \end{vmatrix} = \begin{vmatrix} a & b \\ d & c \end{vmatrix}$$

holds, the determinant $|J|$ is given by $ac - bd$. Furthermore, the trace $trcJ$ is stated by $a + c$. Thus the stability of the equilibrium in tax competition implies $ac - bd$ to be positive and $a + c$ to be negative. To prove, that $a = \frac{\partial^2 u_1}{\partial \tau_1^2}, b = \frac{\partial^2 u_1}{\partial \tau_1 \partial \tau_2}, c = \frac{\partial^2 u_2}{\partial \tau_2^2}$ and $d = \frac{\partial^2 u_2}{\partial \tau_1 \partial \tau_2}$, I derive the equations determining the equilibrium of tax rate decisions. The choice of the tax rate τ_i of region i is given by the first order conditions of the regions:

$$\frac{\partial u_i}{\partial \tau_i} = -k_i - \frac{1}{2}(s_i - ki1) + b'(g_i)\left(k_i + \frac{1}{2}\tau_i \phi'\right) =: 0 \tag{A.40}$$

$$\tag{A.41}$$

Differentiation of (A.40) with respect to both tax rates yields for region 1

$$\begin{aligned}\frac{\partial^2 u_1}{\partial {\tau_1}^2} &= -\frac{1}{2}\phi' + \frac{1}{4}\phi' + b'(g_1)\left(\frac{1}{2}\phi' + \frac{1}{2}\phi'\right) + b''(g_1)\left(k_1 + \frac{1}{2}\tau_1\phi'\right)^2 \\ &= -\frac{1}{4}\phi' + b''(g_1)\left(k_1 + \frac{1}{2}\tau_1\phi'\right)^2 + b'(g_1)\phi' \\ &= a\end{aligned}$$

for the differentiation with respect to τ_1 and

$$\begin{aligned}\frac{\partial^2 u_1}{\partial \tau_1 \partial \tau_2} &= \frac{1}{4}\phi' - b''(g_1)\left(k_1 + \frac{1}{2}\tau_1\phi'\right)\left(\frac{1}{2}\tau_1\phi'\right) + b'(g_1)\left(-\frac{1}{2}\right)\phi' \\ &= b\end{aligned}$$

for the differentiation with respect to τ_2. The differentiation is analog for region 2, so that

$$\begin{aligned}\frac{\partial^2 u_2}{\partial {\tau_2}^2} &= -\frac{1}{4}\phi' + b''(g_2)\left(k_2 + \frac{1}{2}\tau_2\phi'\right)^2 + b'(g_2)\phi' \\ &= c\end{aligned}$$

for the differentiation with respect to τ_2 and

$$\begin{aligned}\frac{\partial^2 u_2}{\partial \tau_2 \partial \tau_1} &= \frac{1}{4}\phi' - b''(g_2)\left(k_2 + \frac{1}{2}\tau_2\phi'\right)\left(\frac{1}{2}\tau_2\phi'\right) + b'(g_2)\left(-\frac{1}{2}\right)\phi' \\ &= d\end{aligned}$$

for the differentiation with respect to τ_1. Thus $|J| = ac - bd$ holds and the determinant of the Jacobian matrix equals the denominators of $(A.36)$ and

$(A.33)$. As this Nash equilibrium of tax rates is assumed to be stable, the $ac - bd > 0$ is proved. Since a and c are both negative, $a + c < 0$ is true and the trace is negative as required in a stable equilibrium.

With $c + b < 0$ and $a + d < 0$

$$\frac{\partial\bar{\tau}_1}{\partial\tilde{\sigma}} = \frac{c+b}{ac-bd} < 0$$
$$\frac{\partial\bar{\tau}_2}{\partial\tilde{\sigma}} = \frac{-(a+d)}{ac-bd} > 0$$

is proved. For the disagreement outcome this implies $\bar{\tau}_1 < \bar{\tau}_2$ in the equilibrium of tax competition (**result II**). Together with $\frac{\partial u_i}{\partial\bar{\tau}_j} > 0, i \neq j$, the sign of X is determined:

$$X = \frac{\partial\bar{u}_1}{\partial\bar{\tau}_2}\cdot\frac{\partial\bar{\tau}_2}{\partial\tilde{\sigma}} - \frac{\partial\bar{u}_2}{\partial\bar{\tau}_1}\cdot\frac{\partial\bar{\tau}_1}{\partial\tilde{\sigma}} > 0.$$

If, in addition, $\bar{r} > r^*$ holds, equation (A.27) is true and $u_1^* - \bar{u}_1 < u_2^* - \bar{u}_2$ is proved.

Since $\frac{\partial r}{\partial\tau_i} = -\frac{1}{2}, i = 1, 2$, it is sufficient to prove $r^* > \bar{r}$ with $\tau_1^* + \tau_2^* > \bar{\tau}_1 + \bar{\tau}_2$. As the model deals only with small asymmetries, both tax rates in the disagreement outcome are inefficiently low, so that the marginal utility $b'(\bar{g}_i) > 1, i = 1, 2$. In W^*, in contrast, the amount of public spending is welfare maximizing and $b'(\bar{g}_i) = 1, i = 1, 2$, holds, so that the level of public spending is higher in both regions. A higher sum of public spending goes along with a higher sum of tax rates, when regions are on the left side of the Laffers curve. Thus $\tau_1^* + \tau_2^* > \bar{\tau}_1 + \bar{\tau}_2$ must be true and $\bar{r} > r^*$ holds. With it, equation (A.27) is true, so that $u_1^* - \bar{u}_1 < u_2^* - \bar{u}_2$ is proved (**result III**).

Proof 9: Ration of Tax Rate in the Nash Bargaining Solution. To determine the ratio of the tax rates, the first order condition given in equation (14) is rearranged to $b'(g_1) = \alpha b'(g_2)$ and the equations (15) and

(16) are inserted, so that

$$\frac{k_1 + \frac{1}{2}(1-\alpha)(s_1 - k_1)}{k_1 + \frac{1}{2}(\tau_1 - \tau_2)\phi'} = \frac{\alpha k_2 + \frac{1}{2}(1-\alpha)(s_1 - k_1)}{k_2 + \frac{1}{2}(\tau_2 - \tau_1)\phi'}$$

is true. Substitute $\frac{1}{2}(1-\alpha)(s_1 - k_1)$ by e and $\frac{1}{2}(\tau_1 - \tau_2)\phi'$ by f to get

$$\Rightarrow \frac{k_1 + e}{k_1 + f} = \frac{\alpha k_2 + e}{k_2 - f}$$
$$\Leftrightarrow k_1 k_2 - k_1 f + k_2 e - ef = \alpha k_1 k_2 + \alpha k_2 f + k_1 e + ef$$
$$\Leftrightarrow (1-\alpha) k_1 k_2 - (k_1 + \alpha k_2) f + (k_2 - k_1) e - 2ef = 0.$$

For $\tau_1 \geq \tau_2$, the capital allocation is given by $k_1 \leq k_2$ and e is positive ($e > 0$), because regions 1 is a capital exporter. Furthermore, f is negative or zero ($f \leq 0$). With $e > 0, \alpha < 1$ and $f \leq 0$ the first term is positive and the last three terms are either also positive or zero. Therefore they cannot sum up to zero. Thus $\tau_1 \geq \tau_2$ is disproved and $\tau_1 < \tau_2$ holds in the Nash bargaining solution.

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